LOW-COST NONLINEAR CONTROL

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Abstract: The paper treats a simple SISO control tool that uses classical nonlinear controllers. Description of basic functions and programs and several examples of their use comprise the main part of the paper. The tool forms part of bigger system programmed in C++ and C# for industrial use. The MATLAB models of the control software and controller adjustment algorithms are freely available on the Czech Technical University web pages. *Copyright © 2007 IFAC*

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1. INTRODUCTION

The Department of Control Engineering of the Faculty of Electrical Engineering of the Czech Technical University in Prague faced in last three years a task of designing a simple and cheap control system for heavy industrial use.

The system is based on several levels of control theory – from adaptive control based on floating horizon to simple classical PID controllers. This paper treats the lowest theoretical level, including analysis of economy of such design.

MATLAB models of the controllers and some auxiliary programs for controller adjustment (John 2005) are available at http://dce.felk.cvut.cz/sri2/ifac.

2. PID CONTROLLER

For case of simplicity, the classical controllers are designed as continuous ones (implemented by digital technique, of course). This decision was based on the fact that modern computers are so fast and the controlled plants so slow that we can afford very short sampling times and the signals in case of rapid changes in the plant behaviour are not delayed by waiting on coming sample period. Also the adjustment of the controllers is very simple for the plant specialists accustomed to analogue controllers.

2.1 Continuous output controllers

Symbolic SIMULINK scheme of continuous output controller is on Fig. 1. The scheme follows the configuration of analogue controllers with central operational amplifier. The saturation of the central amplifier prevents the windup and prolongs the derivative action in case of output saturation – see Fig. 2.

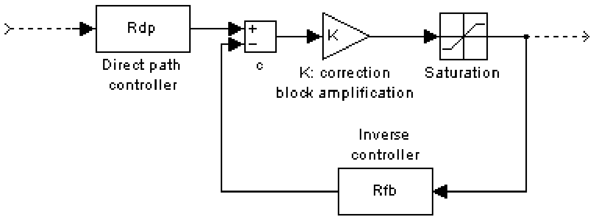

Fig. 1. Symbolic scheme of continuous output controller

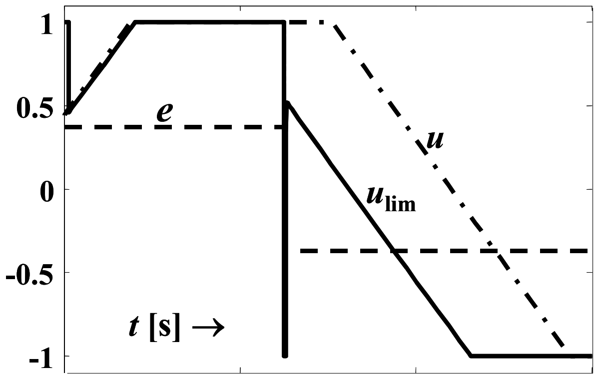

Fig. 2. Effects of central amplifier saturation.

The figure compares output u of strictly linear PID controller with final output saturation ±1 and output u_{lim} of controller from Fig. 1. Controller input e consists of two steps (up and down). Output signal u_{lim} contains two rectangular pulses that replace the Dirac impulses of the controller derivative channel. (The Dirac impulses of linear controller are cut off by the output signal limitation.). Other effect of central member limitation is elimination of windup – in the moment of input signal reversion the nonlinear controller reacts immediately, the linear output is delayed by windup.

PID controller from Fig. 1 can have for example P controller in direct path

$$R_{dp}(s) = K_i \tag{1}$$

and inverse PID controller in the feedback

$$R_{fb}(s) = \frac{s}{(T_i s + 1)(T_d s + 1)} \quad . \tag{2}$$

If the correction gain $K \to \infty$, the overall transfer function of the controller is

$$R_{dp}(s) = K_i \frac{(T_i s + 1)(T_d s + 1)}{s} \quad . \tag{3}$$

Controller defined by (Eqs. 1, 2, 3) has interaction of constants. Our controller is more sophisticated – it is designed as parallel with separate P, I and D channels with structure similar to Fig. 1 and auxiliary filters. Parallel scheme eliminates the problems of interaction

2.2 Pulse controllers

If the saturation in Fig. 1 is replaced by three- or two-level element with adequate hysteresis pulse-width modulated (PWM) PID autopulsative controller is obtained. Typical step response of such controller is on Fig. 3.

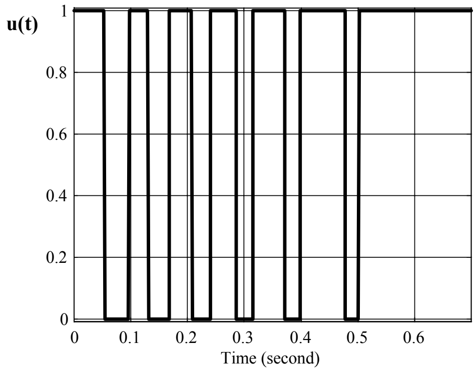

Fig. 3. Step response of PID autopulsative controller

The minimum pulse width is given by hysteresis and correction gain K. For PD controller with derivative time constant T_d and "infinitely short" sampling period the minimum pulse width T_{min} and maximum pulsation frequency f_{max} can be calculated as in Fig. 4. The exponential feedback signal x which is entering the nonlinear element passes the hysteresis h approximately in time given by the slope of a tangent of the exponential. The maximum slope of the exponential

$$x(t) = K \cdot \left(1 - \exp\left(-\frac{t}{T_d}\right)\right) \tag{4}$$

is for $t = 0$ $\dot{x}(0) = K/h$, and consequently

$$T_{\min} = \frac{h \cdot T_d}{t} \quad . \tag{5}$$

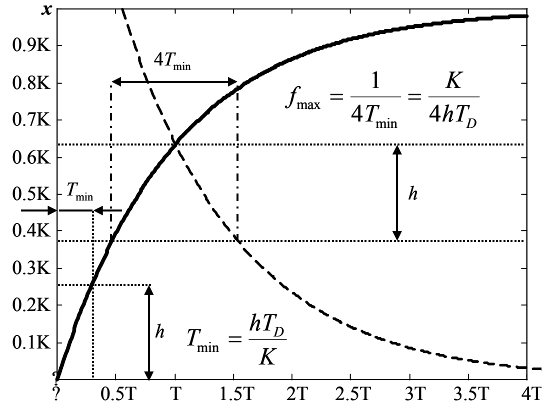

Fig. 4. Calculation of the minimum pulse width

The minimum pulse width is very important for systems with minimum necessary reaction time. For example, asynchronous motor needs several (three to five) electric network periods for a short move.

2.3 Impulse controllers

Impulse controllers or controllers with frequency modulation are used in industry as controllers for step motors or for control of systems with impulse inputs. Typical use is in batch dosing of chemicals: an impulse on the controller output means adding a constant dose of chemical to a mix.

Our controller is not designed for high frequencies of impulses. The maximum pulse frequency is theoretically given by the sampling. So, fast step motors must be driven by some external device.

3. EXAMPLES

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All examples in this paper are programmed as continuous (in MATLAB and Simulink – see http://dce.felk.cvut.cz/sri2/ifac/cont) for the sake of simplicity. More complicated discrete MATLAB models of our programs can be found at http://dce.felk.cvut.cz/sri2/ifac/disc. The mentioned programs in C++ and C# form part of bigger project and belong to the grantor - Czech Ministry of Industry and Commerce.

3.1 Linear continuous output controller

Plant with transfer function

$$S(s)=\frac{100}{(100s+1)\cdot(s+1)^2} \tag{6}$$

is controlled by strictly linear PID controller. The output of the controller is limited by ±1. The controller is adjusted according to the vertical strip pole placement (VSPP) method (John 2005) to obtain relative dampings $a_1 = 0.5$, $a_2 = 1$ to

$$R(s)=\frac{1.25\,(s+0.72)\,(s+0.28)}{s} \quad . \tag{7}$$

Resulting closed-loop transfer function is (see Fig. 5)

$$F(s)=\frac{1.25\,(s+0.72)\cdot(s+0.28)}{(s+0.5)^2\cdot(s^2+s+1)} \quad . \tag{8}$$

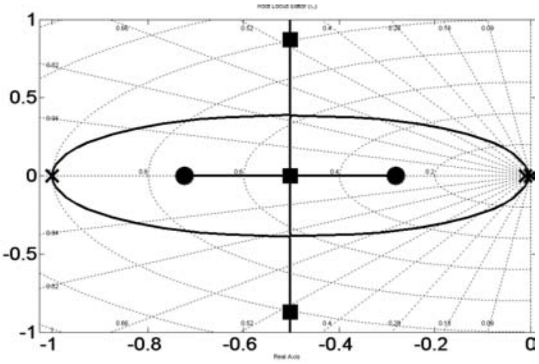

Fig. 5. Root locus of the control circuit.

For setpoint value step $w = 25$ the step response of the control circuit is slowed down by the windup delay – see Fig. 6.

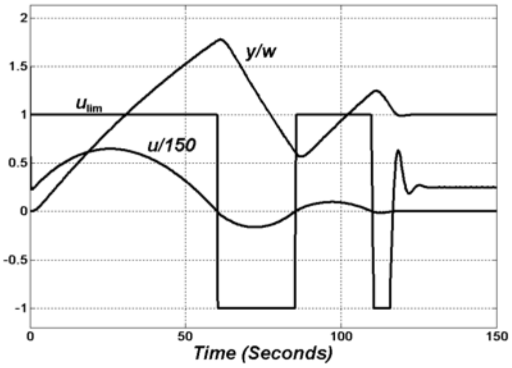

Fig. 6. Strictly linear control with output saturation

3.2 Saturated continuous output controller

If the strictly linear PID controller in the control circuit from the previous paragraph is replaced by a controller from Fig. 1, the step response in the same conditions is much faster and stable – see Fig 7.

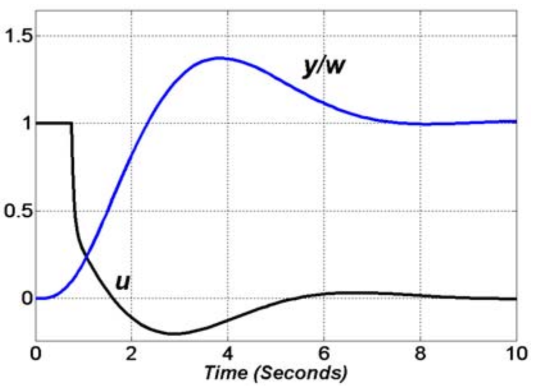

Fig. 7. Control with controller central member limitation

3.3 Three–level PD controller with astatic plant

Astatic plant with transfer function

$$S(s)=\frac{0.01}{s\cdot(100s+1)^2} \tag{9}$$

is controlled by three-level autopulsative controller with feedback PD correction. The controller is adjusted according to VSPP method to obtain relative damping of the resulting control circuit $a = 0.5$. Simulink model of the controller is on Fig. 8. Output levels of the three-level controller are –1; 0; +1, dead zone ±1, hysteresis $h = 1.3$, correction gain $K = 10$, direct path controller $R_{\mathrm{dp}} = 1.185$, inverse controller

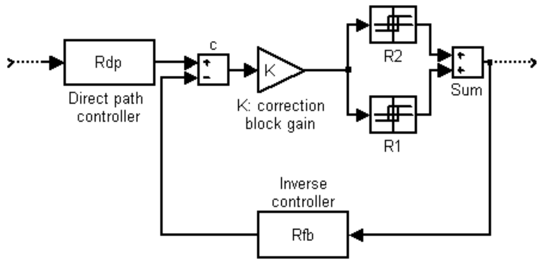

Fig. 8. Three – level PD controller with auto-pulsation

$$R_{fb}(s)=\frac{1}{140s+1} \quad . \tag{10}$$

Minimum pulse length of the controller follows from (1) $T_{\min} = 18.3$s. Dead zone of the controller is $\pm 1/(R_{\mathrm{dp}}\cdot K) = \pm 0.084$. Minimum pulse length satisfies fully the requirements of a simple squirrel-cage asynchronous motor and 50Hz network. In case of necessity, K can be augmented for example up to 183 from which follows dead zone ±0.0046 and $T_{\min} = 1$s. In all cases the sampling period must be at least five times shorter than $T_{\min}$ in order to prevent significant frequency beats.

The resulting transient function of the control circuit with setpoint value step $w = 25$ is on Fig. 9. Relatively small overshot is caused by limitation of controller output (big part of the transient – approximately from 300 to 1100 seconds runs with constant speed).

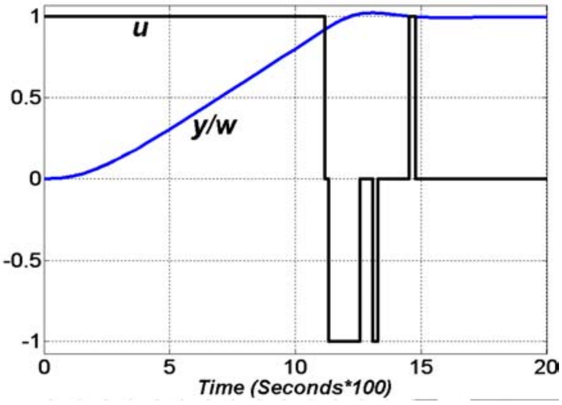

Fig. 9. Transient of the three-level control circuit

Some interesting results can be obtained by changing the feedback correction gain K and hysteresis h.

3.4. Two–level PDD^2 controller with unstable plant

This example is not typical for the control system intended for industrial use, in spite of it shows the possibilities of it. The unstable plant (inverse pendulum) is characterized by transfer function

$$S(s)=\frac{1}{(0.1s+1)^2\cdot(s^2-1)} \quad . \qquad (11)$$

It should be controlled by a two-level PDD2 auto-pulsative controller adjusted to aperiodicity limit.

The VSPP program (*pkpp* from the Czech name) calculates the controller

$$R(s)=7.25\cdot(0.12s+1)\cdot(0.6s+1) \quad . \qquad (12)$$

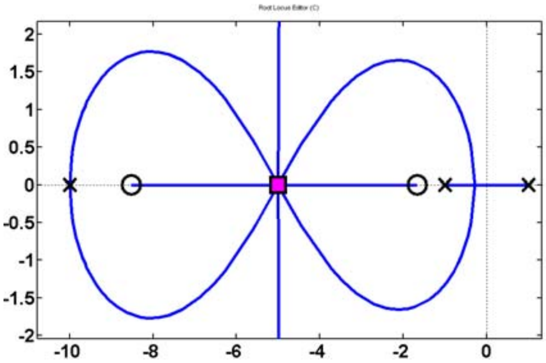

Fig. 10. Root locus for plant (11) with controller (12)

The corresponding root locus is on Fig. 10.

Closed loop transfer function is

$$F(s)=\frac{1.16\cdot(0.12s+1)\cdot(0.6s+1)}{(0.2s+1)^4}=\frac{51\cdot(s+8.3)\cdot(s+0.17)}{(s+5)^4} \quad . \qquad (13)$$

The non-unity gain 1.16 is reached due to negative constant in the plant transfer function denominator. It can be eliminated by proper election of controller constant in the direct path of the control circuit.

Block scheme of the control circuit is on Fig. 11.

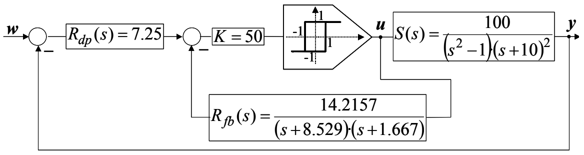

Fig. 11. Block scheme of the control circuit.

The controller output must oscillate to maintain the constant average position of the pendulum. Oscillation amplitude and frequency may be studied by means of describing function program[1], which also belongs to the system – see Figs. 12 and 13.

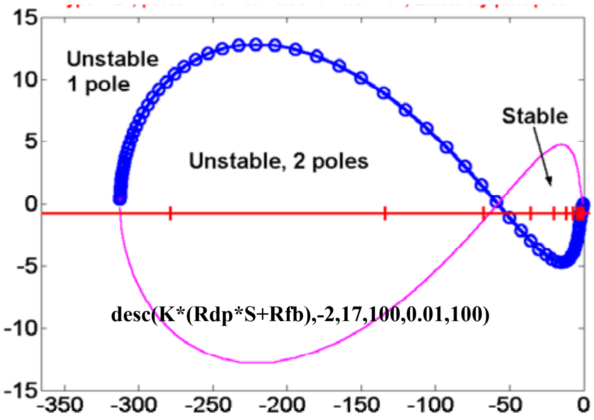

Fig. 12. Describing function with linear Nyquist plot.

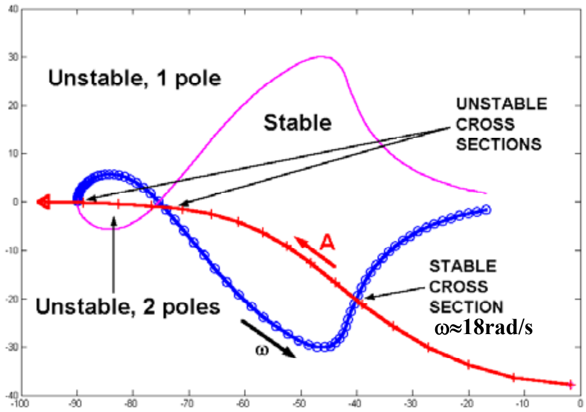

Fig. 13. Describing function with logarithmic Nyquist plot (vector lengths in dB).

Circles mark Nyquist curve and crosses mark inverse negative describing function in both figures. Stable and unstable regions of the Nyquist criterion are marked in the figures. In stable region the amplitude of oscillation decreases, in unstable ones increases, from which follow stable and unstable cross sections,

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[1] Describing function $N(A,\omega)$ is defined as ratio of first harmonics of the output of nonlinear element excited by $A\cdot\cos(\omega t)$ and the exciting amplitude A. If nonlinear system consisting of one nonlinearity and one linear element with transfer function $F(j\omega)$ (upper limiting filter) oscillates with amplitude A (on output of F) at frequency ω, equation $F(j\omega)\cdot N(A,\omega) = -1 \Rightarrow F(j\omega) = -1/N(A,\omega)$.is fulfilled, i.e. $-1/N(A,\omega)$ corresponds to the point –1 in the Nyquist criterion . Figs. 12 and 13 show graphical forms of the equation .$F(j\omega) = -1/N(A,\omega)$.

corresponding to stable and unstable limit cycles. There is only one stable limit cycle in the figures, corresponding to circular frequency $\omega \approx 18$rad/s and amplitude $A \approx 2.09$ on the nonlinear element input – see simulation experiment on Fig. 14. (Amplitude of oscillation on the plant output y is of course much smaller, in our case $A_y \approx 0.0485$.)

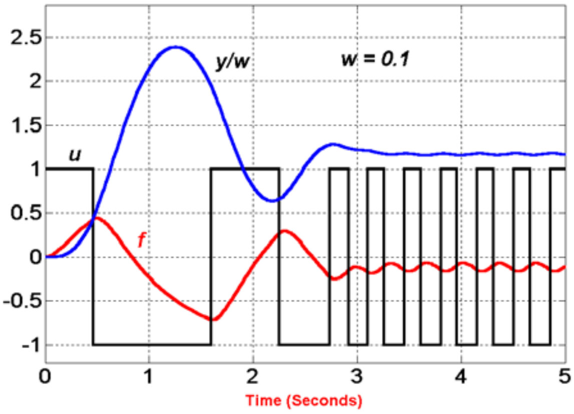

Fig. 14. Stable transient and limit cycle. *f* is auxiliary feedback variable.

Other two crossing points in the figures correspond to unstable limit cycles, one with two poles on both sides and the other for transition from two unstable poles to one unstable pole – see Fig. 15.

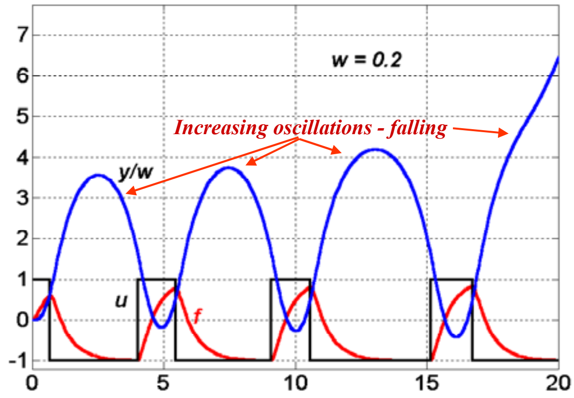

Fig. 15. Unstable transient and limit cycle terminating with crossing to one unstable pole region. *f* is auxiliary feedback variable.

3.5. Three–level PID controller with static plant

The following example shows a control of temperature of three - phase electrical furnace by switching over zero (0) – star (Y) - delta (Δ). The advantage of such control is elimination of higher harmonics in the power network caused by phase – shift continuous control.

Transfer function of the furnace is

$$S(s) = \frac{Y(s)}{U(s)} = \frac{120}{(600s+1)^3} \quad \left[\frac{°\mathrm{C}}{\mathrm{kW}}\right] \quad . \quad (14)$$

The levels of controller output are:
$L_1(0) = 0$kW, $L_2(Y) = 1$kW and $L_2(\Delta) = 3$kW.

The "linear – output" controller is in our case calculated to obtain phase margin 45° and with ratio of time constants $Td/Ti = 10$ (program *frpid*). Its transfer function is

$$R(s) = \frac{U(s)}{E(s)} = \frac{1.37\cdot 10^{-5}\cdot(3460s+1)\cdot(346s+1)}{s} \quad \left[\frac{\mathrm{kW}}{°\mathrm{C}}\right] \quad . \quad (15)$$

Fig. 16 shows the Simulink model of the nonlinear controller. Nonlinear compensation of unequal steps is added in the real control system. The direct path controller is $R_{dp} = 1.37\cdot 10$-5. feedback gain $K = 2\cdot 10^4$, hysteresis of the internal controller (R_1, R_2) is $h = 0.9$. (Outer controller belongs already to the plant model and its levels are $L_1 = 0$, $L_2 = 1$kW, $L_1 = 3$kW.) Feedback controller is

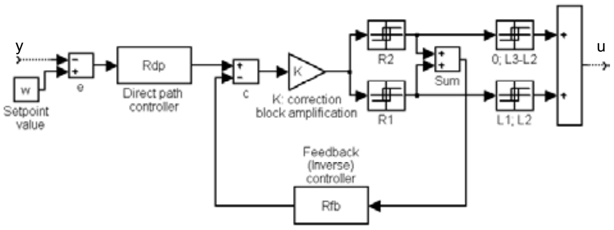

Fig. 16. Simulink model of three – level controller with general output levels L_1, L_2, L_3.

$$R_{fb}(s) = \frac{s}{(3460s+1)\cdot(346s+1)} \quad . \quad (16)$$

Typical transient of the control circuit is on Fig. 17. The controller must pulsate to maintain the desired temperature, because the plant is static. For first thousand seconds the controller heats the furnace by Δ, then for some 1500 seconds disconnects the heating and after two pulses from Y to D continues pulsating between Y and 0. In spite of integral action there is a little constant error in the transient. It is caused by dead zone of the controller: $D_z = \pm 1/(Rdp\cdot K) = \pm 3.7$°C.

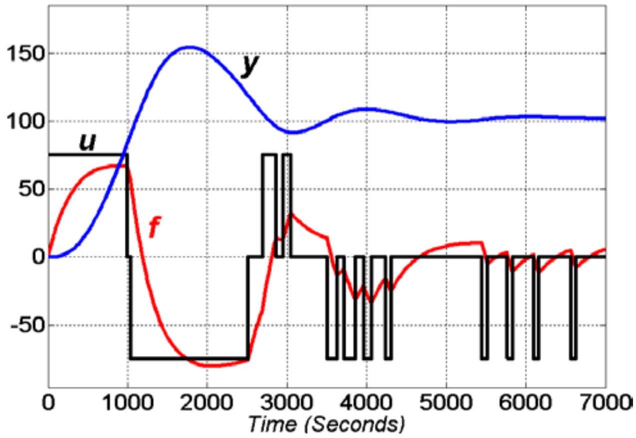

Fig. 17. Control system transient for setpoint value step $w = 100$°C; *y* is plant output, *f* auxiliary variable.

In real control system this disadvantage is eliminated by serial connection of linear PI controller with three – level PD. (The PI controller is there protected against windup.)

4. ECONOMICAL ASPECTS

The major part of savings in control system project is reached by use of simple and cheap elements. Comparing e.g. step motor for driving a control throttle vs. a simple asynchronous squirrel cage motor, one immediately sees the economical advantage of the other. (The problem of precision must be considered, of course.) The simplest drive for typical industrial use is two-phase asynchronous motor with starting capacitor C and two control switches – see Fig. 18.

If upper switch is on, "horizontal" winding is connected via the capacitor, the phase of its voltage is accelerated and the motor turns right (and vice versa). The small inductance L protects the switches against abrupt capacitor discharge in case of connecting both switches at the same time. If there are end limit switches necessary, they can be connected in series with the main control ones. On the other hand, some motors are designed to be used only with mechanical end limit stops and they withstand the possible overload.

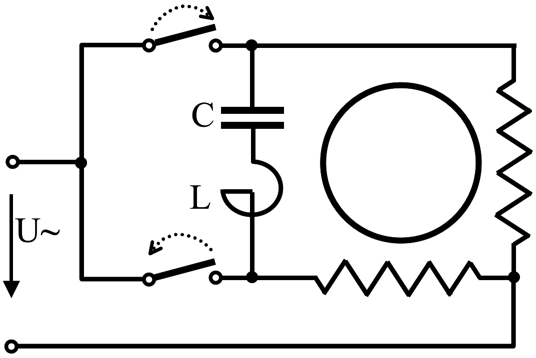

Fig. 18. Two-phase squirrel cage motor with starting capacitor C.

There is another protection for servo drives in our system: if the control signal moves the motor in one direction for longer time than corresponds surely to the end position of the servo, auxiliary signal is generated. This signal is used for blocking the corresponding switch and if the situation endures for prescribed longer time, alert signal is generated. This way ensures not only the servo but also the controlled plant if anything goes wrong.

Another advantage of on-off control of servos is eliminating of friction. Continuous output control of motors may cause problems in small movements of the servos because with small input signal the servo does not move. On the other hand, just disconnect the motor may bring problems with uncontrolled movement by inertia. The drive on Fig. 18 eliminates this problem because it brakes if disconnected, being excited from the capacitor.

Also the other switching control methods may bring some economical advantages – less energy consumption, elimination of higher harmonics problem, etc.

5. CONCLUSIONS

A part of the project of economical control system for industrial use is presented in this paper. The corresponding software may be divided into three groups:

a. MATLAB programs for controller adjustment and control circuit simulation. These programs serve as free courseware for control engineering education (John, 2005; John, 2001).
b. MATLAB programs for simulation of discrete nonlinear control systems and methods for system identification and controller adjustment. Also these programs are available freely on the Web.
c. Ready programs in C++ and C# for industrial use. This part was not complete during the writing of this paper and belongs to grantor.

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